constant-coefficient results are of importance also for such cases, and nonlinear problems are not mentioned. Also, the section on elliptic problems is shorter and more sketchy than would be indicated by the importance of such problems. Here more attention is paid to the iterative solution of the resulting system of linear algebraic equations than the finite difference equations themselves, but even so, there is no mention of modern fast methods such as fast Fourier transform methods or multigrid methods.

One could question the usefulness, for the intended purpose, of a text with the composition of the present book at a time when a large fraction of the computations in engineering and science are done by finite elements, and when also those that are carried out by finite differences relate to problems with variable coefficients, nonlinearities, and complicated geometries. It appears to the reviewer that it is not natural to expect students of the type to which the book is directed to take two consecutive courses on this rather restricted part of the numerical PDE area.

Although the approach of the presentation is novel in some instances, most of the material dates back 15–20 years (more care could have been taken to give a correct historical account). The text contains a lot of nice mathematics, and for someone who was involved in finite differences at the time, it brings back nice memories. The presentation is clear and well organized, and although the reviewer feels that the selection of material is nonoptimal for the purposes stated, and not quite up to date, the book offers a good way of learning what it covers.

V. T.

26[73-02, 65-02, 73K30, 73M05, 65N30, 45L10].—T. A. Cruse, *Boundary Element Analysis in Computational Fracture Mechanics*, Mechanics: Computational Mechanics, Vol. 1, Kluwer Academic Publishers, Dordrecht, 1988, xiii + 162 pp., 24 $\frac{1}{2}$ cm. Price $67.00/Dfl.125.00.

This book presents a collection and review of the author's fundamental contributions to the boundary element method with special emphasis on fracture mechanics. This specific point of view is the fascination of this book as well as its limitation. It is well written and self-contained. From fracture mechanics as well as from boundary integral methods, only those aspects are selected which are of principal importance for computational boundary element methods and which stem from the original work of the author. Hence, the topics are restricted to static fracture mechanics and to classical stationary plasticity. Dynamical aspects and dynamic viscoplasticity are not dealt with. As the author emphasizes, this is not a reference book but an interesting monograph which can serve as an introduction to the field as well as a source of interesting details for the specialist. The mathematical analysis is classical; neither modern variational techniques and results are treated, nor is asymptotic error analysis. The discretizations here are restricted to collocation methods. However, many numerical results illustrate the strength and also the limitation of these boundary element methods.

The first chapter is devoted to historical remarks on boundary integral methods and the boundary element method. This introduction is particularly set by the author's view. I am convinced that any mathematical existence proof which is constructive can always be converted into a computational algorithm—contrary to the author's belief. In the brief review on boundary integral equations, I could not find the enormous contribution by C. F. Gauss, who already in 1839 used the Fredholm integral equation of the first kind to prove existence of the solution of the Dirichlet problem for the Laplacian. (The proof was completed by Feller in the twenties.) Also I could not find references to the thorough analysis and solution algorithm for second-kind Fredholm equations and convex regions by C. Neumann (hundred years ago), to the relation of the boundary integral equation method with the method of reduction to the boundary, founded by Sobolev in the thirties, which started the modern theory of pseudodifferential operators on manifolds; I miss the contributions by F. Noether to the foundation of Cauchy singular boundary integral equations in two, and by Mikhlin, in three dimensions with their applications to elasticity problems. The numerical treatment of boundary integral equations has a longer history than mentioned in the historical chapter. An early version, the panel method used in flow problems, goes back to Lavrentyev in 1932 and was already used for large-scale computations in flow problems and acoustics by Hess and Smith in 1967 and in electromagnetic field computations by Poggio and Miller in 1973. On the other hand, I have learned a lot about the history of boundary integral equations in elasticity from this chapter, which I appreciated very much.

Chapter 2 gives a brief introduction to the boundary value problems for, and the local behavior of, ideal elastic fields near crack tips and crack edges for simplest crack geometries. This includes the Williams expansion and special stress intensity factors.

The third chapter introduces the construction of solutions via boundary potentials defined in terms of fundamental solutions for isotropic as well as anisotropic elastic materials. Somigliana's identity is carefully exemplified as well as the so-called direct and indirect formulations of boundary integral equations. A brief introduction to piecewise polynomial boundary elements and analytic and numerical integration completes this chapter. Mesh refinement at crack tips is heuristically introduced, but mesh grading related to the singular behavior of the stress field is not systematically analyzed.

Chapter 4 deals with the modelling of the crack within the boundary element analysis. Since the hypersingular formulation is here avoided, the direct formulation requires a domain decomposition which imbeds the crack into an interior boundary. The relation between the stress intensity factor and the energy release rate provides a method whose boundary displacement approximations are not related to the specific crack tip behavior. For a better approximation at the crack tips, quarter-point elements and the augmentation method are used.

In Chapter 5, the fundamental solution is chosen to be the Green function for the whole exterior to a straight crack. Then the boundary integral equation on the crack degenerates to a representation formula and is to be solved only on the exterior boundary. Now, the singular behavior of stress and displacement as well as the stress intensity factors are completely inherent in the Betti representation formula. This method proved to be most accurate and efficient. Here it is also presented for anisotropic materials. For piecewise linear boundary elements, the integrations are performed analytically.

Chapter 6 contains the extension of the method to elastoplastic crack problems by introducing artificial forcing terms corresponding to the nonlinear stress contributions in the integral equations. As long as these terms are small—which corresponds to small regions of plasticification—they can be treated iteratively. Here, also an incremental approach is used. The additional volume integrals for piecewise linear displacements on triangulations of the plasticity domain are evaluated analytically. The resulting algorithm is extremely fast, and the numerical results for test problems are excellent; they are compared with finite element results obtained with ADINA.

In Chapter 7, the displacement discontinuity method leads to the hypersingular boundary integral equations on the crack. As is known from recent analysis, the hypersingular boundary integral operator defines a pseudodifferential operator of order $+1$, hence, the collocation boundary element method can only be conforming for continuously differentiable trial functions. For such elements, polar coordinates and integration by parts in the radial direction is used for the evaluation of the hypersingular integrals. At the crack tip in two, and crack edge in three dimensions, the special form of the singularities is incorporated into the trial spaces. The resulting numerical scheme provides very convincing results for elliptic cracks in three dimensions.

In Chapter 8, the boundary element method of Chapter 5 is modified for being utilized by the weight function method of Bueckner and Rice with the necessary weight functions belonging to the specific two-dimensional crack problem. The strength of the method is demonstrated for three reference problems.

This book will surely find many friends among advanced students, and also among researchers in mechanics and in numerical and applied mathematics.

W. WENDLAND

Mathematisches Institut A
Universität Stuttgart
Pfaffenwaldring 57
D-7000 Stuttgart 80
West Germany